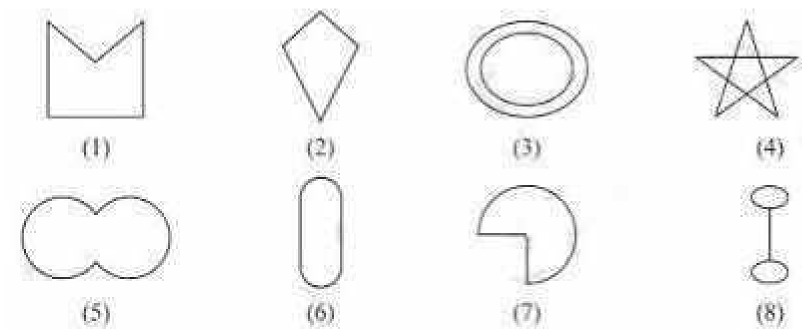
CHAPTER 12

QUADRILATERALS

1. Given here are some figures.



Classify each of them on the basis of the following.

Simple curve (b) Simple closed curve (c) Polygon

(d) Convex polygon (e) Concave polygon

Solution:

a) Simple curve: 1, 2, 5, 6 and 7

b) Simple closed curve: 1, 2, 5, 6 and 7

c) Polygon: 1 and 2

d) Convex polygon: 2

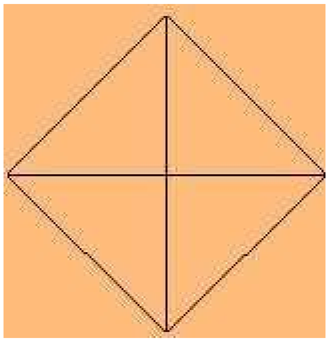
e) Concave polygon: 1

2. How many diagonals does each of the following have?

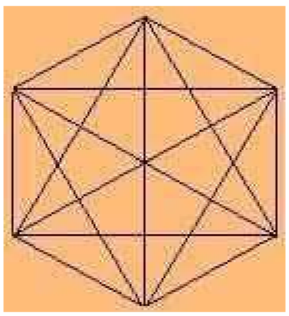
a) A convex quadrilateral (b) A regular hexagon (c) A triangle

Solution:

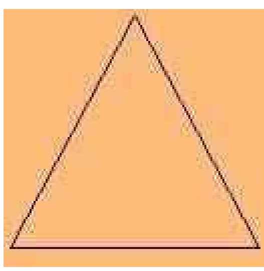
a) A convex quadrilateral: 2.



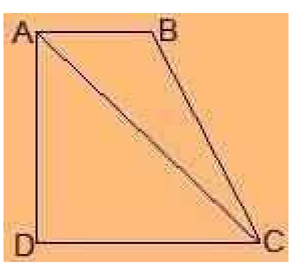
b) A regular hexagon: 9.



c) A triangle: 0



3. What is the sum of the measures of the angles of a convex quadrilateral? Will this property hold if the quadrilateral is not convex? (Make a non-convex quadrilateral and try!) Solution:



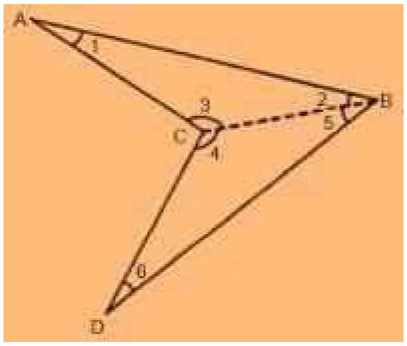
Let ABCD be a convex quadrilateral.

From the figure, we infer that the quadrilateral ABCD is formed by two triangles,

i.e. ΔADC and ΔABC.

Since we know that sum of the interior angles of a triangle is 180°, the

sum of the measures of the angles is 180° + 180° = 360°



Let us take another quadrilateral ABCD which is not convex .

Join BC, such that it divides ABCD into two triangles ΔABC and ΔBCD. In ΔABC,

∠1 + ∠2 + ∠3 = 180° (angle sum property of triangle)

In ΔBCD,

∠4 + ∠5 + ∠6 = 180° (angle sum property of triangle)

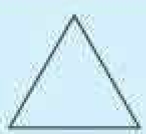
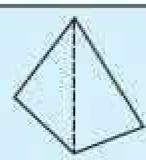
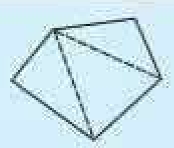
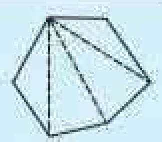
∴, ∠1 + ∠2 + ∠3 + ∠4 + ∠5 + ∠6 = 180° + 180°

⇒ ∠1 + ∠2 + ∠3 + ∠4 + ∠5 + ∠6 = 360°

⇒ ∠A + ∠B + ∠C + ∠D = 360°

Thus, this property holds if the quadrilateral is not convex.

4. Examine the table. (Each figure is divided into triangles and the sum of the angles deduced from that.)

Figure				
Side	3	4	5	6
Angle sum	180°	2 × 180° = (4 – 2) × 180°	3 × 180° = (5 – 2) × 180°	4 × 180° = (6 – 2) × 180°

What can you say about the angle sum of a convex polygon with number of sides? (a) 7 (b) 8 (c) 10 (d) n Solution:

The angle sum of a polygon having side n = $(n-2)\times180°$ a)

7

Here, n = 7

Thus, angle sum = $(7-2)\times180° = 5\times180° = 900°$

b) 8

Here, n = 8

Thus, angle sum = $(8-2)\times180° = 6\times180° = 1080°$

c) 10

Here, n = 10

Thus, angle sum = $(10-2)\times180° = 8\times180° = 1440°$

d) n

Here, n = n

Thus, angle sum = $(n-2)\times180°$

5. What is a regular polygon?

State the name of a regular polygon of

(i) 3 sides (ii) 4 sides (iii) 6 sides Solution:

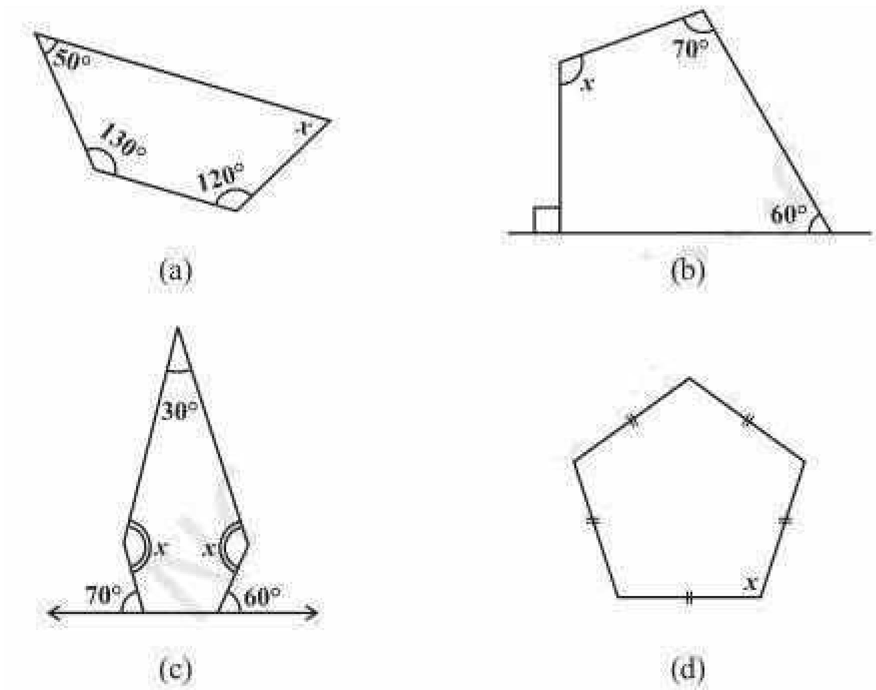
Regular polygon: A polygon having sides of equal length and angles of equal measures is called a regular polygon. A regular polygon is both equilateral and equiangular.

(i) A regular polygon of 3 sides is called an equilateral triangle.

(ii) A regular polygon of 4 sides is called a square.

(iii) A regular polygon of 6 sides is called a regular hexagon.

6. Find the angle measure of x in the following figures.



Solution:

a) The figure has 4 sides. Hence, it is a quadrilateral. Sum of angles of the quadrilateral = 360° ⇒ 50° + 130° + 120° + x = 360°

⇒ 300° + x = 360° ⇒ x

= 360° – 300° = 60°

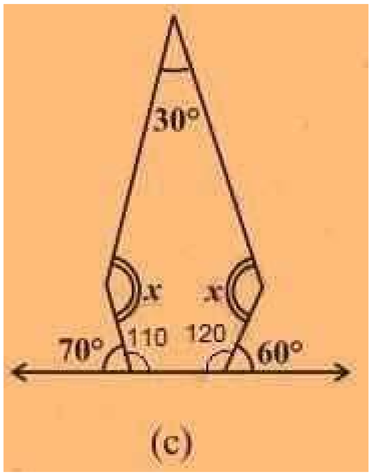
b) The figure has 4 sides. Hence, it is a quadrilateral. Also, one side is perpendicular forming a right angle. Sum of angles of the quadrilateral = 360°

⇒ 90° + 70° + 60° + x = 360°

⇒ 220° + x = 360° ⇒ x =

360° – 220° = 140°

c) The figure has 5 sides. Hence, it is a pentagon.



Sum of angles of the pentagon = 540° Two angles at the bottom are a linear pair.

∴, 180° – 70° = 110°

180° – 60° = 120°

⇒ 30° + 110° + 120° + x + x = 540°

⇒ 260° + 2x = 540°

⇒ 2x = 540° – 260° = 280°

⇒ 2x = 280°

= 140°

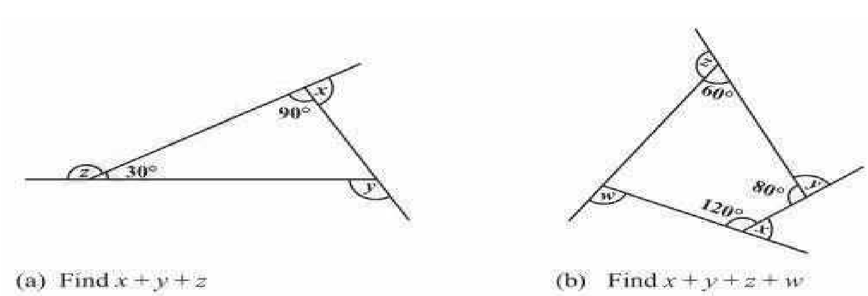
d) The figure has 5 equal sides. Hence, it is a regular pentagon. Thus, all its angles are equal.

5x = 540°

⇒ x = 540°/5

⇒ x = 108°

7.



Solution:

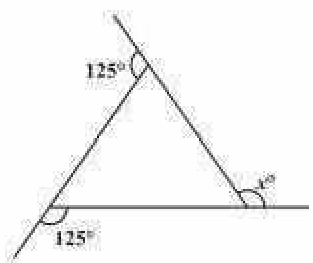
a) Sum of all angles of triangle = 180° One side of triangle = 180°- (90° + 30°) = 60° x + 90° = 180° ⇒ x = 180° – 90° = 90° y + 60° = 180° ⇒ y = 180° – 60° = 120° z + 30° = 180° ⇒ z = 180° – 30° = 150° x + y + z = 90° + 120° + 150° = 360°

b) Sum of all angles of quadrilateral = 360°

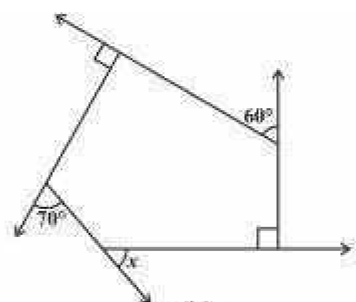
One side of quadrilateral = 360°- (60° + 80° + 120°) = 360° – 260° = 100°

x + 120° = 180° ⇒ x = 180° – 120° = 60° y + 80° = 180° ⇒ y = 180° – 80° = 100° z + 60° = 180° ⇒ z = 180° – 60° = 120° w + 100° = 180° ⇒ w = 180° – 100° = 80° x + y + z + w = 60° + 100° + 120° + 80° = 360°

8. Find x in the following figures.



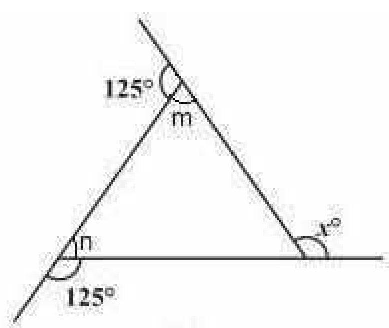
(a)



(b)

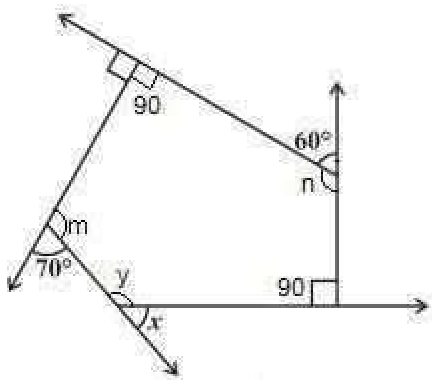
Solution:

a)



$125° + m = 180° \Rightarrow m = 180° – 125° = 55°$ (Linear pair) $125° + n = 180° \Rightarrow n = 180° – 125° = 55°$ (Linear pair) $x = m + n$ (The exterior angle of a triangle is equal to the sum of the two opposite interior angles) $\Rightarrow x = 55° + 55° = 110°$

b)



Two interior angles are right angles = 90°

$70° + m = 180° \Rightarrow m = 180° – 70° = 110°$ (Linear pair)

$60° + n = 180° \Rightarrow n = 180° – 60° = 120°$ (Linear pair) The figure is having five sides and is a pentagon.

Thus, sum of the angles of a pentagon = 540°

$\Rightarrow 90° + 90° + 110° + 120° + y = 540°$

$\Rightarrow 410° + y = 540° \Rightarrow y = 540° – 410° = 130°$

$x + y = 180°$ (Linear pair) $\Rightarrow x + 130° = 180°$

$\Rightarrow x = 180° – 130° = 50°$

9. Find the measure of each exterior angle of a regular polygon of

(i) 9 sides (ii) 15 sides

Solution:

Sum of the angles of a regular polygon having side $n = (n-2)\times180°$

(i) Sum of the angles of a regular polygon having 9 sides $= (9-2)\times180° = 7\times180° = 1260°$

Each interior angle $= 1260/9 = 140°$

Each exterior angle $= 180° – 140° = 40°$

Or,

Each exterior angle = Sum of exterior angles/Number of angles = 360/9 = 40°

(ii) Sum of angles of a regular polygon having side 15 = (15-2)×180°

= 13×180° = 2340°

Each interior angle = 2340/15 = 156°

Each exterior angle = 180° – 156° = 24°

Or,

Each exterior angle = sum of exterior angles/Number of angles = 360/15 = 24°

10.How many sides does a regular polygon have if the measure of an exterior angle is 24°?

Solution:

Each exterior angle = sum of exterior angles/Number of angles

24°= 360/ Number of sides ⇒

Number of sides = 360/24 = 15

Thus, the regular polygon has 15 sides.

11. How many sides does a regular polygon have if each of its interior angles is 165°?

Solution:

Interior angle = 165°

Exterior angle = 180° – 165° = 15°

Number of sides = sum of exterior angles/exterior angles

⇒ Number of sides = 360/15 = 24

Thus, the regular polygon has 24 sides.

12. a) Is it possible to have a regular polygon with measure of each exterior angle as 22°?

b) Can it be an interior angle of a regular polygon? Why?

Solution:

a) Exterior angle = 22°

Number of sides = sum of exterior angles/ exterior angle

⇒ Number of sides = 360/22 = 16.36

No, we can't have a regular polygon with each exterior angle as 22° as it is not a divisor of 360. b) Interior angle = 22°

Exterior angle = 180° – 22°= 158°

No, we can't have a regular polygon with each exterior angle as 158° as it is not a divisor of 360.

13. a) What is the minimum interior angle possible for a regular polygon? Why?

b) What is the maximum exterior angle possible for a regular polygon?

Solution:

a) An equilateral triangle is the regular polygon (with 3 sides) having the least possible minimum interior angle because a regular polygon can be constructed with minimum 3 sides.

Since the sum of interior angles of a triangle = 180°

Each interior angle = 180/3 = 60°

b) An equilateral triangle is the regular polygon (with 3 sides) having the maximum exterior angle because the regular polygon with the least number of sides has the maximum exterior angle possible. Maximum exterior possible = 180 – 60° = 120°