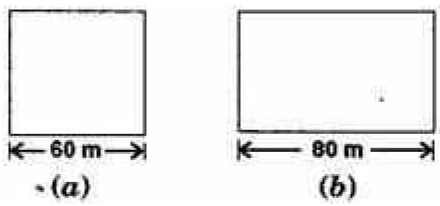
CHAPTER 14

MENSURATION

1. A square and a rectangular field with measurements as given in the figure have the same perimeter. Which field has a larger area?



Solution:

Side of a square = 60 m (Given)

And the length of the rectangular field, l = 80 m (Given)

According to the question,

The perimeter of the rectangular field = Perimeter of the square field

2(l+b) = 4×Side (using formulas)

2(80+b) = 4×60

160+2b = 240 b

= 40

The breadth of the rectangle is 40 m.

Now, the Area of the Square field

= $(\text{side})^2$

= $(60)^2 = 3600\ m^2$

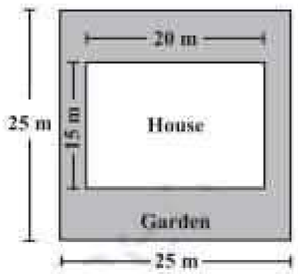
And the Area of the Rectangular field

= length×breadth = 80×40

= $3200\ m^2$

Hence, the area of the square field is larger.

2. Mrs.Kaushik has a square plot with the measurement as shown in the figure. She wants to construct a house in the middle of the plot. A garden is developed around the house. Find the total cost of developing a garden around the house at the rate of Rs. 55 per m^2.



Solution:

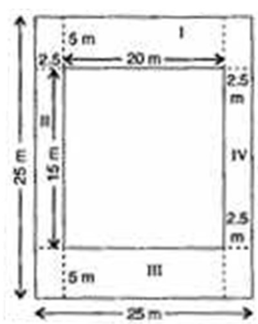
Side of the square plot = 25 m

Formula: Area of the square plot = square of the side = $(\text{side})^2$

= $(25)^2$ = 625

Therefore the area of the square plot is 625 m^2

Length of the house = 20 m and



The breadth of the house = 15 m

Area of the house = length×breadth

= 20×15 = 300 m^2

Area of the garden = Area of the square plot – Area of the house

= 625–300 = 325 m^2

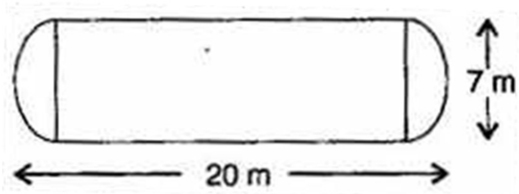
∵ The cost of developing the garden per sq. m is Rs. 55

The cost of developing the garden 325 sq. m = Rs. 55×325

= Rs. 17,875

Hence the total cost of developing a garden is around Rs. 17,875.

3. The shape of a garden is rectangular in the middle and semi-circular at the ends as shown in the diagram. Find the area and the perimeter of this garden [Length of rectangle is 20 – (3.5 + 3.5 meters]



Solution::

Given: Total length = 20 m

Diameter of the semi-circle = 7 m

Radius of the semi-circle = 7/2 = 3.5 m

Length of the rectangular field

= 20-(3.5+3.5) = 20-7 = 13 m

Breadth of the rectangular field = 7 m

Area of rectangular field = l×b

= 13×7= 91m^2

Area of the two semi-circles = 2×(1/2)×π×r^2

= 2×(1/2)×22/7×3.5×3.5

= 38.5 m^2

Area of garden = 91+38.5 = 129.5 m^2

Now, the perimeter of the two semi-circles = 2πr = 2×(22/7)×3.5 = 22 m

And the perimeter of the garden = 22+13+13

= 48 m. Answer

4. A flooring tile has the shape of a parallelogram whose base is 24 cm and the corresponding height is 10 cm. How many such tiles are required to cover a floor of area 1080 m^2? [If required you can split the tiles in whatever way you want to fill up the corners] Solution:

Given: Base of flooring tile = 24 cm = 0.24 m

Corresponding height of a flooring tile= 10 cm = 0.10 m

Now Area of flooring tile= Base×Altitude

= 0.24×0.10

= 0.024

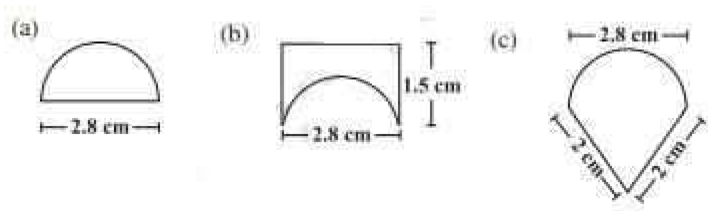
Area of flooring tile is $0.024 m^2$

Number of tiles required to cover the floor= Area of floor/Area of one tile = 1080/0.024 =

45000 tiles

Hence 45000 tiles are required to cover the floor.

5. An ant is moving around a few food pieces of different shapes scattered on the floor. For which food-piece would the ant have to take a longer round? Remember, circumference of a circle can be obtained by using the expression $C = 2\pi r$,where r is the radius of the circle.



Solution:

(a) Radius = Diameter/2 = 2.8/2 cm = 1.4 cm

Circumference of semi-circle = πr

= (22/7)×1.4 = 4.4

Circumference of the semi-circle is 4.4 cm

Total distance covered by the ant= Circumference of semi -circle+Diameter

= 4.4+2.8 = 7.2 cm

(b) Diameter of semi-circle = 2.8 cm

Radius = Diameter/2 = 2.8/2 = 1.4 cm

Circumference of semi-circle = r

= (22/7)×1.4 = 4.4 cm

Total distance covered by the ant= 1.5+2.8+1.5+4.4 = 10.2 cm

(c) Diameter of semi-circle = 2.8 cm

Radius = Diameter/2 = 2.8/2

= 1.4 cm

Circumference of semi-circle = πr

= (22/7)×1.4

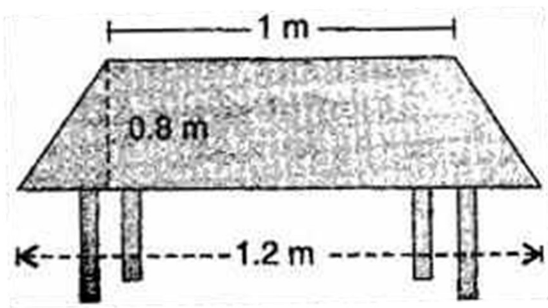
= 4.4 cm

Total distance covered by the ant = 2+2+4.4 = 8.4 cm

After analyzing the results of three figures, we concluded that for figure (b) food piece, the ant would take a longer round.

EXTRA QUESTION

1.The shape of the top surface of a table is a trapezium. Find its area if its parallel sides are 1 m and 1.2 m and perpendicular distance between them is 0.8 m.



Solution: One parallel side of the trapezium (a) = 1 m

And second side (b) = 1.2 m and height (h) = 0.8 m

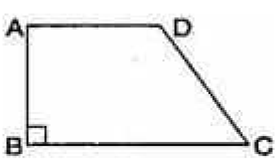
Area of top surface of the table= (½)×(a+b)h

= (½)×(1+1.2)0.8

= (½)×2.2×0.8 = 0.88

Area of top surface of the table is 0.88 m^2 .

2. The area of a trapezium is 34 $cm^2$and the length of one of the parallel sides is 10 cm and its height is 4 cm Find the length of the other parallel side.



Solution: Let the length of the other parallel side be b.

Length of one parallel side, a = 10 cm height, (h) = 4

cm and

Area of a trapezium is 34 cm^2

Formula for, Area of trapezium = (1/2)×(a+b)h

34 = ½(10+b)×4

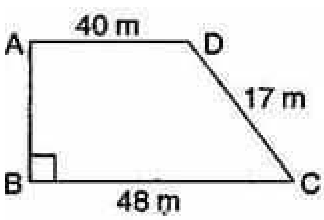
34 = 2×(10+b)

After simplifying, b = 7

Hence another required parallel side is 7 cm.

3. Length of the fence of a trapezium shaped field ABCD is 120 m. If BC = 48 m, CD = 17 m and AD = 40 m, find the area of this field. Side AB is perpendicular to the parallel sides AD and BC.

Solution:



Given: BC = 48 m, CD = 17 m,

AD = 40 m and perimeter = 120 m

∵ Perimeter of trapezium ABCD

= AB+BC+CD+DA

120 = AB+48+17+40

120 = AB = 105

AB = 120–105 = 15 m

Now, Area of the field= (½)×(BC+AD)×AB

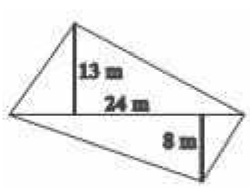
= (½)×(48 +40)×15

= (½)×88×15

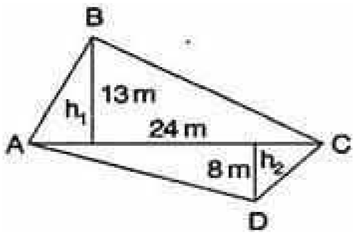
= 660

Hence, area of the field ABCD is 660m^2.

4. The diagonal of a quadrilateral shaped field is 24 m and the perpendiculars dropped on it from the remaining opposite vertices are 8 m and 13 m. Find the area of the field.



Solution:



Consider, h_1 = 13 m, h_2 = 8 m and AC = 24 m

Area of quadrilateral ABCD = Area of triangle ABC+Area of triangle ADC

$= ½(bh_1)+ ½(bh_2)$

$= ½ \times b(h_1+h_2)= (½)\times 24\times(13+8)$

$= (½)\times 24\times 21 = 252$

Hence, the required area of the field is 252 m^2

5. The diagonals of a rhombus are 7.5 cm and 12 cm. Find its area.

Solution:

Given: d1 = 7.5 cm and d2 = 12 cm

We know that the area of a rhombus = (½)×d1×d2

= (½)×7.5×12 = 45

Therefore, the area of the rhombus is 45 cm^2 .

6. Find the area of a rhombus whose side is 5 cm and whose altitude is 4.8 cm. If one of the diagonals is 8 cm long, find the length of the other diagonal.

Solution: Since a rhombus is also a kind of parallelogram,

The formula for Area of rhombus = Base×Altitude

Putting values, we have

Area of rhombus = 6×4 = 24

Area of rhombus is 24 cm^2

Also, Formula for Area of rhombus = $(½)\times d_1 d_2$

After substituting the values, we get

$24 = (½)×8×d_2$ d_2

$= 6$

Hence, the length of the other diagonal is 6 cm.

7. The floor of a building consists of 3000 tiles which are rhombus shaped and each of its diagonals are 45 cm and 30 cm in length. Find the total cost of polishing the floor, if the cost per m^2 is Rs. 4.

Solution:

Length of one diagonal, $d_1 = 45$ cm and $d_2 = 30$ cm

∵ Area of one tile = $(½)d_1d_2$ =

$(½)×45×30 = 675$

Area of one tile is 675 cm^2

Area of 3000 tiles is

= 675×3000 = 2025000 cm^2

= 2025000/10000

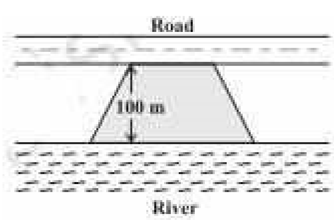
= 202.50 m^2 [∵ $1m^2 = 10000\ cm^2$]

∵ Cost of polishing the floor per sq. meter = 4

Cost of polishing the floor per 202.50 sq. meter = 4×202.50 = 810

Hence the total cost of polishing the floor is Rs. 810.

8. Mohan wants to buy a trapezium shaped field. Its side along the river is parallel to and twice the side along the road. If the area of this field is 10500 m^2 and the perpendicular distance between the two parallel sides is 100 m, find the length of the side along the river.



Solution:

Perpendicular distance (h) = 100 m (Given)

Area of the trapezium-shaped field = 10500 m^2 (Given)

Let the side along the road be 'x' m and the side along the river = 2x m

Area of the trapezium field = $(\frac{1}{2})\times(a+b)\times h$

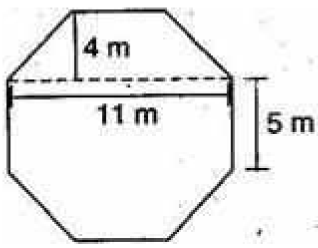
$10500 = (\frac{1}{2})\times(x+2x)\times 100$

$10500 = 3x\times 50$

After simplifying, we have $x = 70$, which means the side along the river is 70 m

Hence, the side along the river = $2x = 2(70) = 140$ m.

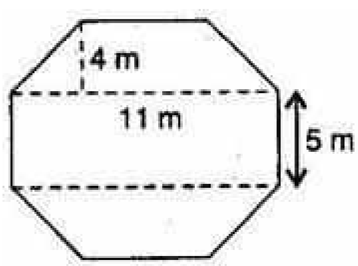
9. Top surface of a raised platform is in the shape of a regular octagon as shown in the figure. Find the area of the octagonal surface.



Solution:

The octagon has eight equal sides, each 5 m. (given)

Divide the octagon as shown in the below figure, 2 trapeziums whose parallel and perpendicular sides are 11 m and 4 m respectively and 3rd one is rectangle having length and breadth 11 m and 5 m respectively.



Now, Area of two trapeziums = $2[(\frac{1}{2})\times(a+b)\times h]$

$= 2\times(\frac{1}{2})\times(11+5)\times 4$

$= 4\times 16 = 64$

Area of two trapeziums is 64 m^2

Also, Area of rectangle = length×breadth

$= 11\times 5 = 55$

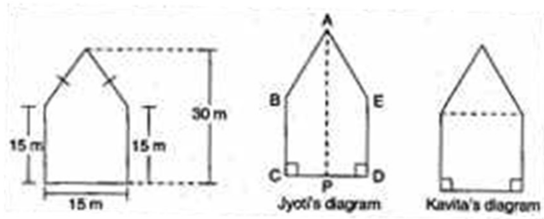
Area of rectangle is 55 m^2

Total area of octagon = 64+55

$= 119\ m^2$

10. There is a pentagonal shaped park as shown in the figure.

For finding its area Jyoti and Kavita divided it in two different ways.



Find the area of this park using both ways. Can you suggest some other way of finding its area?

Solution:

First way: By Jyoti's diagram,

Area of pentagon = Area of trapezium ABCP + Area of trapezium AEDP

$= (½)(AP+BC)\times CP+(1/2)\times(ED+AP)\times DP$

$= (½)(30+15)\times CP+(1/2)\times(15+30)\times DP$

$= (½)\times(30+15)\times(CP+DP)$

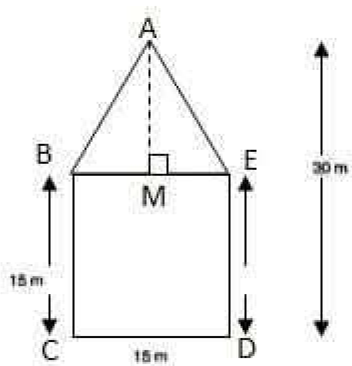
$= (½)\times 45\times CD$

$= (1/2)\times 45\times 15$

$=337.5\ m^2$

Area of pentagon is $337.5\ m^2$

Second way: By Kavita's diagram



Here, a perpendicular AM is drawn to BE.

AM = 30–15 = 15 m

Area of pentagon = Area of triangle ABE+Area of square BCDE (from above figure)

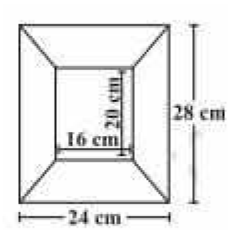
= (½)×15×15+(15×15)

= 112.5+225.0

= 337.5

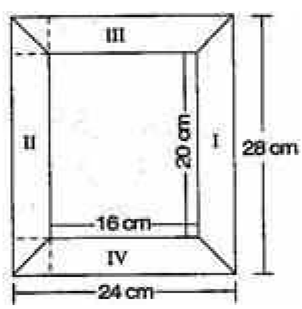
Hence, the total area of pentagon-shaped park = 337.5 m^2

11. Diagram of the adjacent picture frame has outer dimensions = 24 cm×28 cm and inner dimensions 16 cm×20 cm. Find the area of each section of the frame, if the width of each section is same.



Solution:

Divide given figure into 4 parts, as shown below:



Here two of the given figures (I) and (II) are similar in dimensions.

And also, figures (III) and (IV) are similar in dimensions.

Area of figure (I) = Area of trapezium

= (½)×(a+b)×h

= (½)×(28+20)×4

= (½)×48×4 = 96

Area of figure (I) = 96 cm^2

Also, Area of figure (II) = 96 cm^2

Now, Area of figure (III) = Area of trapezium

$= (\frac{1}{2})\times(a+b)\times h$

$= (\frac{1}{2})\times(24+16)4$

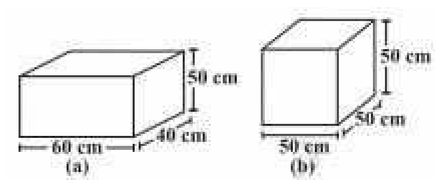
$= (\frac{1}{2})\times 40\times 4 = 80$

Area of figure (III) is 80 cm^2

Also, Area of figure (IV) = 80 cm^2

EXTRA QUESTION 2

1. There are two cuboidal boxes as shown in the adjoining figure. Which box requires the lesser amount of material to make?



Solution:

(a) Given: Length of cuboidal box (l) = 60 cm

Breadth of cuboidal box (b) = 40 cm

Height of cuboidal box (h) = 50 cm

Total surface area of cuboidal box = $2\times(lb+bh+hl)$

$= 2\times(60\times40+40\times50+50\times60)$

$= 2\times(2400+2000+3000)$

= 14800 cm^2

(b) Length of cubical box (l) = 50 cm

Breadth of cubicalbox (b) = 50 cm

Height of cubicalbox (h) = 50 cm

Total surface area of cubical box = $6(\text{side})^2$

$= 6(50\times50)$

$= 6\times2500$

= 15000

Surface area of the cubical box is 15000 cm^2

From the result of (a) and (b), cuboidal box requires the lesser amount of material to make.

2. A suitcase with measures 80 cm x 48 cm x 24 cm is to be covered with a tarpaulin cloth. How many meters of tarpaulin of width 96 cm is required to cover 100 such suitcases?

Solution:

Length of suitcase box, l = 80 cm,

Breadth of suitcase box, b= 48 cm

And Height of cuboidal box , h = 24 cm

Total surface area of suitcase box = 2(lb+bh+hl)

= 2(80×48+48×24+24×80)

= 2 (3840+1152+1920)

= 2×6912

= 13824

Total surface area of suitcase box is 13824 cm^2

Area of Tarpaulin cloth = Surface area of suitcase

l×b = 13824 l ×96 = 13824 l = 144

Required tarpaulin for 100 suitcases = 144×100 = 14400 cm = 144 m

Hence tarpaulin cloth required to cover 100 suitcases is 144 m.

3. Find the side of a cube whose surface area is 600cm^2 .

Solution:

Surface area of cube = 600 cm^2 (Given)

Formula for surface area of a cube = $6(\text{side})^2$

Substituting the values, we get

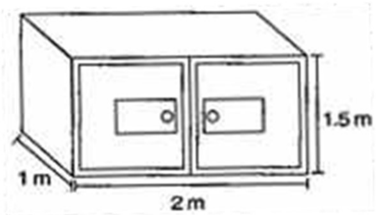
$6(\text{side})^2 = 600$

$(\text{side})^2 = 100$

Or side = ±10

Since side cannot be negative, the measure of each side of a cube is 10 cm

4. Rukshar painted the outside of the cabinet of measure 1 m ×2 m ×1.5 m. How much surface area did she cover if she painted all except the bottom of the cabinet?



Solution:

Length of cabinet, l = 2 m, Breadth of cabinet, b = 1 m and Height of cabinet, h = 1.5 m

Area painted = Total surface area of the cabinet – Area of bottom

Total surface area of the cabinet = 2(lb+bh+hl)

= 2(2×1+1×1.5+1.5×2)

= 2(2+1.5+3.0)

= 13 m^2

Area of bottom = Length × Breadth

= 2 × 1

= 2 m^2

Area painted = 13 – 2 = 11 m^2

The required surface area of the cabinet is 11m^2.

5. Daniel is paining the walls and ceiling of a cuboidal hall with length, breadth and height of 15 m, 10 m and 7 m respectively. From each can of paint 100 m^2of area is painted. How many cans of paint will she need to paint the room?

Solution:

Length of wall, l = 15 m, Breadth of wall, b = 10 m and Height of wall, h = 7 m

Total Surface area of classroom = lb+2(bh+hl)

= 15×10+2(10×7+7×15)

= 150+2(70+105)

= 150+350

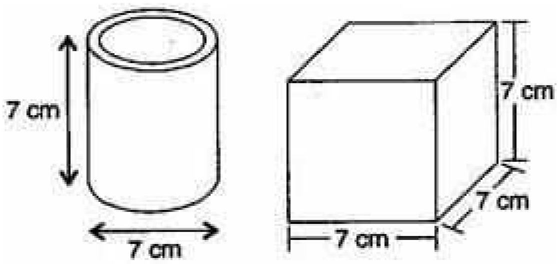
= 500

Now, Required number of cans = Area of hall/Area of one can

= 500/100 = 5

Therefore, 5 cans are required to paint the room.

6. Describe how the two figures below are alike and how they are different. Which box has larger lateral surface areas?



Solution:

Similarity

Both figures have the same length and the same height

Difference

The first figure has circular bottom and top

The second figure has square bottom and top

The first figure is a cylinder and the second figure is a cube

Diameter of cylinder = 7 cm (Given)

Radius of cylinder, r = 7/2 cm

Height of cylinder, h = 7 cm

Lateral surface area of cylinder = $2\pi rh$

= $2\times(22/7)\times(7/2)\times7 = 154$

So, Lateral surface area of cylinder is 154 cm^2

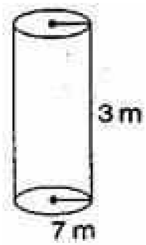
Now, lateral surface area of cube = $4\ (side)^2 = 4\times7^2 = 4\times49 = 196$

Lateral surface area of cube is 196 cm^2

Hence, the cube has a larger lateral surface area.

7. A closed cylindrical tank of radius 7 m and height 3 m is made from a sheet of metal. How much sheet of metal is required?

Solution:



Radius of cylindrical tank, r = 7 m

Height of cylindrical tank , h = 3 m

Total surface area of cylindrical tank = 2πr(h+r)

= 2×(22/7)×7(3+7)

= 44×10 = 440

Therefore, 440 m^2 metal sheet is required.

8. The lateral surface area of a hollow cylinder is 4224cm^2. It is cut along its height and formed a rectangular sheet of width 33 cm. Find the perimeter of rectangular sheet?

Solution:

Lateral surface area of hollow cylinder = 4224 cm^2

Width of rectangular sheet = 33 cm and say l be the length of the rectangular sheet

Lateral surface area of cylinder = Area of the rectangular sheet

4224 = b × l 4224 =

33 × l l = 4224/33 =

128 cm

So the length of the rectangular sheet is 128 cm.

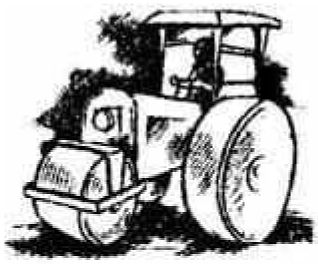
Also, Perimeter of rectangular sheet = 2(l+b)

= 2(128+33)

= 322 cm

The perimeter of the rectangular sheet is 322 cm.

9. A road roller takes 750 complete revolutions to move once over to level a road. Find the area of the road if the diameter of a road roller is 84 cm and length 1 m.



Solution:

Diameter of road roller, d = 84 cm

Radius of road roller, r = d/2 = 84/2 = 42 cm

Length of road roller, h = 1 m = 100 cm

Formula for Curved surface area of road roller = $2\pi rh$

$= 2\times(22/7)\times42\times100 = 26400$

Curved surface area of the road roller is 26400 cm^2

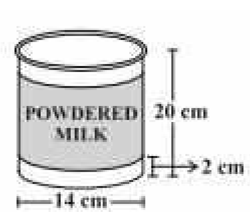
Again, Area covered by the road roller in 750 revolutions = $26400\times750 cm^2$

$= 1,98,00,000 cm^2$

$= 1980\ m^2$ [$\because$ 1 m^2= 10,000 cm^2]

Hence the area of the road is 1980 m^2.

10. A company packages its milk powder in cylindrical container whose base has a diameter of 14 cm and height 20 cm. Company places a label around the surface of the container (as shown in figure). If the label is placed 2 cm from top and bottom, what is the area of the label?



Solution:

Diameter of the cylindrical container , d = 14 cm

Radius of cylindrical container, r = d/2 = 14/2 = 7 cm

Height of cylindrical container = 20 cm

Height of the label, say h = 20–2–2 (from the figure)

= 16 cm

Curved surface area of label = $2\pi rh$

= $2\times(22/7)\times7\times16$

= 704

Hence, the area of the label is 704 cm^2.