

CHAPTER 6

ALGEBRAIC EXPRESSIONS

1. Find the product of the following pairs of monomials.

(i) $4, 7p$

(ii) $-4p, 7p$

(iii) $-4p, 7pq$

(iv) $4p^3, -3p$

(v) $4p, 0$

Solution:

(i) $4, 7p = 4 \times 7 \times p = 28p$

(ii) $-4p \times 7p = (-4 \times 7) \times (p \times p) = -28p^2$

(iii) $-4p \times 7pq = (-4 \times 7) \times (p \times pq) = -28p^2q$

(iv) $4p^3 \times -3p = (4 \times -3) \times (p^3 \times p) = -12p^4$ (v) $4p \times 0 = 0$

2. Find the areas of rectangles with the following pairs of monomials as their lengths and breadths, respectively.

(p, q) ; (10m, 5n) ; (20x², 5y²) ; (4x, 3x²) ; (3mn, 4np)

Solution:

Area of rectangle = Length x breadth. So, it is multiplication of two monomials.

The results can be written in square units.

(i) $p \times q = pq$

(ii) $10m \times 5n = 50mn$

(iii) $20x^2 \times 5y^2 = 100x^2y^2$

(iv) $4x \times 3x^2 = 12x^3$

(v) $3mn \times 4np = 12mn^2p$

3. Complete the following table of products:

First monomial → Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$					
$-5y$			$-15x^2y$			
$3x^2$						
$-4xy$						
$7x^2y$						
$-9x^2y^2$						

Solution:

First monomial	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
Second monomial						
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

4. Obtain the volume of rectangular boxes with the following length, breadth and height, respectively.

- (i) $5a, 3a^2, 7a^4$
(ii) $2p, 4q, 8r$
(iii) $xy, 2x^2y, 2xy^2$
(iv) $a, 2b, 3c$ **Solution:**

Volume of rectangle = length x breadth x height. To evaluate volume of rectangular boxes, multiply all the monomials.

- (i) $5a \times 3a^2 \times 7a^4 = (5 \times 3 \times 7)(a \times a^2 \times a^4) = 105a^7$
(ii) $2p \times 4q \times 8r = (2 \times 4 \times 8)(p \times q \times r) = 64pqr$
(iii) $y \times 2x^2y \times 2xy^2 = (1 \times 2 \times 2)(x \times x^2 \times x \times y \times y \times y^2) = 4x^4y^4$
(iv) $a \times 2b \times 3c = (1 \times 2 \times 3)(a \times b \times c) = 6abc$

5. Obtain the product of

(i) xy, yz, zx

(ii) $a, -a^2, a^3$

(iii) $2, 4y, 8y^2, 16y^3$

(iv) $a, 2b, 3c, 6abc$

(v) $m, -mn, mnp$ **Solution:**

(i) $xy \times yz \times zx = x^2 y^2 z^2$

(ii) $a \times -a^2 \times a^3 = -a^6$

(iii) $2 \times 4y \times 8y^2 \times 16y^3 = 1024 y^6$

(iv) $a \times 2b \times 3c \times 6abc = 36a^2 b^2 c^2$

(v) $m \times -mn \times mnp = -m^3 n^2 p$

EXTRA QUESTION

1. Carry out the multiplication of the expressions in each of the following pairs.

(i) $4p, q + r$

(ii) $ab, a - b$

(iii) $a + b, 7a^2b^2$

(iv) $a^2 - 9, 4a$ (v) $pq + qr + rp, 0$

Solution:

(i) $4p(q + r) = 4pq + 4pr$

(ii) $ab(a - b) = a^2b - ab^2$

(iii) $(a + b)(7a^2b^2) = 7a^3b^2 + 7a^2b^3$

(iv) $(a^2 - 9)(4a) = 4a^3 - 36a$

(v) $(pq + qr + rp) \times 0 = 0$ (Anything multiplied by zero is zero)

2. Complete the table.

	First expression	Second expression	Product
(i)	a	$b + c + d$	
(ii)	$x + y - 5$	$5xy$	
(iii)	p	$6p^2 - 7p + 5$	
(iv)	$4p^2q^2$	$p^2 - q^2$	
(v)	$a + b + c$	abc	

Solution:

	First expression	Second expression	Product
(i)	a	$b + c + d$	$a(b+c+d)$ $= a \times b + a \times c + a \times d$ $= ab + ac + ad$
(ii)	$x + y - 5$	$5xy$	$5xy(x + y - 5)$ $= 5xy \times x + 5xy \times y - 5xy \times 5$ $= 5x^2y + 5xy^2 - 25xy$
(iii)	p	$6p^2 - 7p + 5$	$p(6p^2 - 7p + 5)$ $= p \times 6p^2 - p \times 7p + p \times 5$ $= 6p^3 - 7p^2 + 5p$
(iv)	$4p^2q^2$	$p^2 - q^2$	$4p^2q^2 * (p^2 - q^2)$ $= 4p^4q^2 - 4p^2q^4$
(v)	$a + b + c$	abc	$abc(a + b + c)$ $= abc \times a + abc \times b + abc \times c$ $= a^2bc + ab^2c + abc^2$

3. Find the product.

i) $a^2 \times (2a^{22}) \times (4a^{26})$

$$\text{ii) } (2/3 \ xy) \times (-9/10 \ x^2y^2)$$

$$\text{(iii) } (-10/3 \ pq^3) \times (6/5 \ p^3q)$$

$$\text{(iv) } (x) \times (x^2) \times (x^3) \times (x^4) \text{ Solution:}$$

$$\text{i) } a^2 \times (2a^{22}) \times (4a^{26})$$

$$= (2 \times 4) (a^2 \times a^{22} \times a^{26})$$

$$= 8 \times a_{2+22+26}$$

$$= 8a^{50} \text{ ii) } (2xy/3) \times (-$$

$$9x^2y^2/10)$$

$$= (2/3 \times -9/10) (x \times x^2 \times y \times y^2)$$

$$= (-3/5 \ x^3y^3) \text{ iii) } (-10pq^3/3)$$

$$\times (6p^3q/5)$$

$$= (-10/3 \times 6/5) (p \times p^3 \times q^3 \times q)$$

$$= (-4p^4q^4) \text{ iv) } (x) \times (x^2) \times (x^3) \times$$

$$(x^4)$$

$$= x_{1+2+3+4}$$

$$= x^{10}$$

4. (a) Simplify $3x(4x - 5) + 3$ and find its values for (i) $x = 3$ (ii) $x = 1/2$ (b)

Simplify $a(a^2 + a + 1) + 5$ and find its value for (i) $a = 0$, (ii) $a = 1$ (iii) $a = -1$.

Solution:

$$\text{a) } 3x(4x - 5) + 3$$

$$= 3x(4x) - 3x(5) + 3$$

$$= 12x^2 - 15x + 3$$

$$\text{(i) Putting } x=3 \text{ in the equation we gets } 12x^2 - 15x + 3 = 12(3^2) - 15(3) + 3$$

$$= 108 - 45 + 3$$

$$= 66$$

$$\text{(ii) Putting } x=1/2 \text{ in the equation we get}$$

$$12x^2 - 15x + 3 = 12 (1/2)^2 - 15 (1/2) + 3$$

$$= 12 (1/4) - 15/2 + 3$$

$$= 3 - 15/2 + 3$$

$$= 6 - 15/2$$

$$= (12 - 15) / 2$$

$$= -3/2$$

$$\mathbf{b) \ a(a^2+a+1)+5}$$

$$= a \times a^2 + a \times a + a \times 1 + 5 = a^3 + a^2 + a + 5$$

$$(i) \text{ putting } a=0 \text{ in the equation we get } 0^3+0^2+0+5=5$$

$$(ii) \text{ putting } a=1 \text{ in the equation we get } 1^3+1^2+1+5=1+1+1+5=8$$

$$(iii) \text{ Putting } a = -1 \text{ in the equation we get } (-1)^3+(-1)^2+(-1)+5 = -1+1-1+5=4$$

$$\mathbf{5. (a) \text{ Add: } p(p-q), q(q-r) \text{ and } r(r-p)}$$

$$\mathbf{(b) \text{ Add: } 2x(z-x-y) \text{ and } 2y(z-y-x)}$$

$$\mathbf{(c) \text{ Subtract: } 3l(l-4m+5n) \text{ from } 4l(10n-3m+2l) \quad (d) \text{ Subtract: } 3a(a+b+c)-2$$

$$b(a-b+c) \text{ from } 4c(-a+b+c)}$$

Solution:

$$a) \ p(p-q) + q(q-r) + r(r-p)$$

$$= (p^2 - pq) + (q^2 - qr) + (r^2 - pr)$$

$$= p^2 + q^2 + r^2 - pq - qr - pr$$

$$b) \ 2x(z-x-y) + 2y(z-y-x)$$

$$= (2xz - 2x^2 - 2xy) + (2yz - 2y^2 - 2xy)$$

$$= 2xz - 4xy + 2yz - 2x^2 - 2y^2$$

$$c) \ 4l(10n-3m+2l) - 3l(l-4m+5n) = (40ln - 12lm + 8l^2) - (3l^2 - 12lm + 15ln)$$

$$= 40ln - 12lm + 8l^2 - 3l^2 + 12lm - 15ln$$

$$= 25ln + 5l^2$$

$$d) \ 4c(-a+b+c) - (3a(a+b+c) - 2b(a-b+c))$$

$$= (-4ac + 4bc + 4c^2) - (3a^2 + 3ab + 3ac - (2ab - 2b^2 + 2bc))$$

$$= -4ac + 4bc + 4c^2 - (3a^2 + 3ab + 3ac - 2ab + 2b^2 - 2bc)$$

$$= -4ac + 4bc + 4c^2 - 3a^2 - 3ab - 3ac + 2ab - 2b^2 + 2bc$$

$$= -7ac + 6bc + 4c^2 - 3a^2 - ab - 2b^2$$

6. Multiply the binomials.

(i) $(2x + 5)$ and $(4x - 3)$

(ii) $(y - 8)$ and $(3y - 4)$

(iii) $(2.5l - 0.5m)$ and $(2.5l + 0.5m)$

(iv) $(a + 3b)$ and $(x + 5)$

(v) $(2pq + 3q^2)$ and $(3pq - 2q^2)$ (vi) $(\frac{3}{4}a^2 + 3b^2)$ and $4(\frac{1}{2}a^2 - \frac{2}{3}b^2)$

Solution :

(i) $(2x + 5)(4x - 3)$

$$= 2x \times 4x - 2x \times 3 + 5 \times 4x - 5 \times 3$$

$$= 8x^2 - 6x + 20x - 15$$

$$= 8x^2 + 14x - 15$$

(ii) $(y - 8)(3y - 4)$

$$= y \times 3y - 4y - 8 \times 3y + 32$$

$$= 3y^2 - 4y - 24y + 32$$

$$= 3y^2 - 28y + 32$$

(iii) $(2.5l - 0.5m)(2.5l + 0.5m)$

$$= 2.5l \times 2.5l + 2.5l \times 0.5m - 0.5m \times 2.5l - 0.5m \times 0.5m$$

$$= 6.25l^2 + 1.25lm - 1.25lm - 0.25m^2$$

$$= 6.25l^2 - 0.25m^2$$

$$(a + 3b)(x + 5)$$

$$= ax + 5a + 3bx + 15b$$

$$v) (2pq + 3q^2)(3pq - 2q^2)$$

$$= 2pq \times 3pq - 2pq \times 2q^2 + 3q^2 \times 3pq - 3q^2 \times 2q^2$$

$$= 6p^2q^2 - 4pq^3 + 9pq^3 - 6q^4$$

$$= 6p^2q^2 + 5pq^3 - 6q^4$$

$$(vi) (3/4 a^2 + 3b^2) \text{ and } 4(a^2 - 2/3 b^2)$$

$$= (3/4 a^2 + 3b^2) \times 4(a^2 - 2/3 b^2)$$

$$= (3/4 a^2 + 3b^2) \times (4a^2 - 8/3 b^2)$$

$$= 3/4 a^2 \times (4a^2 - 8/3 b^2) + 3b^2 \times (4a^2 - 8/3 b^2)$$

$$= 3/4 a^2 \times 4a^2 - 3/4 a^2 \times 8/3 b^2 + 3b^2 \times 4a^2 - 3b^2 \times 8/3 b^2$$

$$= 3a^4 - 2a^2 b^2 + 12 a^2 b^2 - 8b^4$$

$$= 3a^4 + 10a^2 b^2 - 8b^4$$

7. Find the product.

$$(i) (5 - 2x)(3 + x)$$

$$(ii) (x + 7y)(7x - y)$$

$$(iii) (a^2 + b)(a + b^2) \quad (iv) (p^2 - q^2)(2p + q) \quad \text{Solution:}$$

$$(i) (5 - 2x)(3 + x)$$

$$= 5(3 + x) - 2x(3 + x)$$

$$= 15 + 5x - 6x - 2x^2$$

$$= 15 - x - 2x^2$$

$$(ii) (x + 7y)(7x - y)$$

$$= x(7x - y) + 7y(7x - y)$$

$$= 7x^2 - xy + 49xy - 7y^2$$

$$= 7x^2 - 7y^2 + 48xy \text{ iii)}$$

$$(a^2 + b)(a + b^2)$$

$$= a^2(a + b^2) + b(a + b^2)$$

$$= a^3 + a^2b^2 + ab + b^3 =$$

$$a^3 + b^3 + a^2b^2 + ab \text{ iv)}$$

$$(p^2 - q^2)(2p + q) = p^2$$

$$(2p + q) - q^2(2p + q)$$

$$= 2p^3 + p^2q - 2pq^2 - q^3 =$$

$$2p^3 - q^3 + p^2q - 2pq^2$$

8. Simplify.

$$(i) (x^2 - 5)(x + 5) + 25$$

$$(ii) (a^2 + 5)(b^3 + 3) + 5$$

$$(iii) (t + s^2)(t^2 - s)$$

$$(iv) (a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$$

$$(v) (x + y)(2x + y) + (x + 2y)(x - y)$$

$$(vi) (x + y)(x^2 - xy + y^2)$$

$$(vii) (1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$$

$$(viii) (a + b + c)(a + b - c) \text{ Solution:}$$

$$i) (x^2 - 5)(x + 5) + 25$$

$$= x^3 + 5x^2 - 5x - 25 + 25$$

$$= x^3 + 5x^2 - 5x \text{ ii) } (a^2 + 5)$$

$$(b^3 + 3) + 5$$

$$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5 =$$

$$a^2b^3 + 5b^3 + 3a^2 + 20$$

$$iii) (t + s^2)(t^2 - s)$$

$$= t(t^2 - s) + s^2(t^2 - s)$$

$$= t^3 - st + s^2t^2 - s^3 = t^3 - s^3 - st + s^2t^2 \text{ iv) } (a + b)(c$$

$$- d) + (a - b)(c + d) + 2(ac + bd) = (a + b)(c$$

$$- d) + (a - b)(c + d) + 2(ac + bd)$$

$$= (ac - ad + bc - bd) + (ac + ad - bc - bd) + (2ac + 2bd)$$

$$= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$$

$$= 4ac$$

$$\text{v) } (x + y)(2x + y) + (x + 2y)(x - y)$$

$$= 2x^2 + xy + 2xy + y^2 + x^2 - xy + 2xy - 2y^2$$

$$= 3x^2 + 4xy - y^2 \text{ vi) } (x$$

$$+ y)(x^2 - xy + y^2)$$

$$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$$

$$= x^3 + y^3 \text{ vii) } (1.5x - 4y)(1.5x + 4y + 3) -$$

$$4.5x + 12y$$

$$= 2.25x^2 + 6xy + 4.5x - 6xy - 16y^2 - 12y - 4.5x + 12y = 2.25x^2 - 16y^2 \text{ viii)}$$

$$(a + b + c)(a + b - c)$$

$$= a^2 + ab - ac + ab + b^2 - bc + ac + bc - c^2$$

$$= a^2 + b^2 - c^2 + 2ab$$

9. Use a suitable identity to get each of the following products.

$$\text{(i) } (x + 3)(x + 3)$$

$$\text{(ii) } (2y + 5)(2y + 5)$$

$$\text{(iii) } (2a - 7)(2a - 7)$$

$$\text{(iv) } (3a - 1/2)(3a - 1/2)$$

$$\text{(v) } (1.1m - 0.4)(1.1m + 0.4)$$

$$\text{(vi) } (a^2 + b^2)(-a^2 + b^2)$$

$$(vii) (6x - 7)(6x + 7)$$

$$(viii) (-a + c)(-a + c)$$

$$(ix) (1/2x + 3/4y)(1/2x + 3/4y)$$

$$(x) (7a - 9b)(7a - 9b)$$

Solution:

$$(i) (x + 3)(x + 3) = (x + 3)^2$$

$$= x^2 + 6x + 9$$

$$\text{Using } (a+b)^2 = a^2 + b^2 + 2ab \text{ ii)}$$

$$(2y + 5)(2y + 5) = (2y + 5)^2 =$$

$$4y^2 + 20y + 25$$

$$\text{Using } (a+b)^2 = a^2 + b^2 + 2ab \text{ iii)}$$

$$(2a - 7)(2a - 7) = (2a - 7)^2 =$$

$$4a^2 - 28a + 49$$

$$\text{Using } (a-b)^2 = a^2 + b^2 - 2ab \text{ iv) } (3a$$

$$- 1/2)(3a - 1/2) = (3a - 1/2)^2$$

$$= 9a^2 - 3a + (1/4)$$

$$\text{Using } (a-b)^2 = a^2 + b^2 - 2ab$$

$$v) (1.1m - 0.4)(1.1m + 0.4)$$

$$= 1.21m^2 - 0.16$$

$$\text{Using } (a - b)(a + b) = a^2 - b^2 \text{ vi)}$$

$$(a^2 + b^2)(-a^2 + b^2)$$

$$= (b^2 + a^2)(b^2 - a^2)$$

$$= -a^4 + b^4$$

$$\text{Using } (a - b)(a + b) = a^2 - b^2 \text{ vii)}$$

$$(6x - 7)(6x + 7)$$

$$= 36x^2 - 49$$

Using $(a - b)(a + b) = a^2 - b^2$ viii)

$$(-a + c)(-a + c) = (-a + c)^2$$

$$= c^2 + a^2 - 2ac$$

Using $(a-b)^2 = a^2 + b^2 - 2ab$

$$\text{ix) } \left(\frac{1}{2}x + \frac{3}{4}y\right)\left(\frac{1}{2}x + \frac{3}{4}y\right) = \left(\frac{1}{2}x + \frac{3}{4}y\right)^2$$

$$= (x^2/4) + (9y^2/16) + (3xy/4)$$

Using $(a+b)^2 = a^2 + b^2 + 2ab$

$$\text{x) } (7a - 9b)(7a - 9b) = (7a - 9b)^2$$

$$= 49a^2 - 126ab + 81b^2$$

Using $(a-b)^2 = a^2 + b^2 - 2ab$

10. Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products.

$$\text{(i) } (x + 3)(x + 7)$$

$$\text{(ii) } (4x + 5)(4x + 1)$$

$$\text{(iii) } (4x - 5)(4x - 1)$$

$$\text{(iv) } (4x + 5)(4x - 1)$$

$$\text{(v) } (2x + 5y)(2x + 3y)$$

$$\text{(vi) } (2a^2 + 9)(2a^2 + 5)$$

$$\text{(vii) } (xyz - 4)(xyz - 2) \text{ Solution:}$$

$$\text{(i) } (x + 3)(x + 7)$$

$$= x^2 + (3+7)x + 21$$

$$= x^2 + 10x + 21 \text{ ii)}$$

$$(4x + 5)(4x + 1)$$

$$= 16x^2 + 4x + 20x + 5$$

$$= 16x^2 + 24x + 5 \text{ iii)}$$

$$(4x - 5)(4x - 1)$$

$$= 16x^2 - 4x - 20x + 5$$

$$= 16x^2 - 24x + 5 \text{ iv)}$$

$$(4x + 5)(4x - 1)$$

$$= 16x^2 + (5-1)4x - 5$$

$$= 16x^2 + 16x - 5$$

$$\text{v)} (2x + 5y)(2x + 3y)$$

$$= 4x^2 + (5y + 3y)2x + 15y^2$$

$$= 4x^2 + 16xy + 15y^2 \text{ vi)}$$

$$(2a^2 + 9)(2a^2 + 5)$$

$$= 4a^4 + (9+5)2a^2 + 45$$

$$= 4a^4 + 28a^2 + 45 \text{ vii)}$$

$$(xyz - 4)(xyz - 2)$$

$$= x^2y^2z^2 + (-4 - 2)xyz + 8$$

$$= x^2y^2z^2 - 6xyz + 8$$

11. Find the following squares by using the identities.

$$\text{(i)} \quad (b - 7)^2$$

$$\text{(ii)} \quad (xy + 3z)^2$$

$$\text{(iii)} \quad (6x^2 - 5y)^2$$

$$\text{(iv)} \quad [(2m/3) + (3n/2)]^2$$

$$\text{(v)} \quad (0.4p - 0.5q)^2$$

$$\text{(vi)} \quad (2xy + 5y)^2 \text{ **Solution:**}$$

Using identities:

$$(a - b)^2 = a^2 + b^2 - 2ab \quad (a + b)^2 = a^2 + b^2 + 2ab$$

$$\text{(i)} \quad (b - 7)^2 = b^2 - 14b + 49$$

$$\text{(ii)} \quad (xy + 3z)^2 = x^2y^2 + 6xyz + 9z^2$$

$$\text{(iii)} \quad (6x^2 - 5y)^2 = 36x^4 - 60x^2y + 25y^2$$

$$\text{(iv)} \quad [(2m/3) + (3n/2)]^2 = (4m^2/9) + (9n^2/4) + 2mn$$

$$(v) (0.4p - 0.5q)^2 = 0.16p^2 - 0.4pq + 0.25q^2$$

$$(vi) (2xy + 5y)^2 = 4x^2y^2 + 20xy^2 + 25y^2$$

12. Simplify.

$$(i) (a^2 - b^2)^2$$

$$(ii) (2x + 5)^2 - (2x - 5)^2$$

$$(iii) (7m - 8n)^2 + (7m + 8n)^2$$

$$(iv) (4m + 5n)^2 + (5m + 4n)^2$$

$$(v) (2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$$

$$(vi) (ab + bc)^2 - 2ab^2c \quad (vii) (m^2 - n^2m)^2 + 2m^3n^2 \quad \text{Solution:}$$

$$i) (a^2 - b^2)^2 = a^4 + b^4 - 2a^2b^2$$

$$ii) (2x + 5)^2 - (2x - 5)^2 \\ = 4x^2 + 20x + 25 - (4x^2 - 20x + 25) = 4x^2 + 20x + 25 - 4x^2 + 20x - 25 = 40x$$

$$iii) (7m - 8n)^2 + (7m + 8n)^2 \\ = 49m^2 - 112mn + 64n^2 + 49m^2 + 112mn + 64n^2 \\ = 98m^2 + 128n^2$$

$$iv) (4m + 5n)^2 + (5m + 4n)^2 = 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2 \\ = 41m^2 + 80mn + 41n^2$$

$$v) (2.5p - 1.5q)^2 - (1.5p - 2.5q)^2 \\ = 6.25p^2 - 7.5pq + 2.25q^2 - 2.25p^2 + 7.5pq - 6.25q^2 \\ = 4p^2 - 4q^2 \quad vi) (ab + bc)^2 - 2ab^2c = a^2b^2 + 2ab^2c + b^2c^2 - 2ab^2c \\ = a^2b^2 + b^2c^2$$

$$vii) (m^2 - n^2m)^2 + 2m^3n^2 = m^4 \\ - 2m^3n^2 + m^2n^4 + 2m^3n^2 = m^4 \\ + m^2n^4$$

13. Show that.

$$(i) (3x + 7)^2 - 84x = (3x - 7)^2$$

$$(ii) (9p - 5q)^2 + 180pq = (9p + 5q)^2$$

$$(iii) (4/3m - 3/4n)^2 + 2mn = 16/9 m^2 + 9/16 n^2$$

$$(iv) (4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$$

$$(v) (a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$$

Solution:

$$\text{i) LHS} = (3x + 7)^2 - 84x$$

$$= 9x^2 + 42x + 49 - 84x$$

$$= 9x^2 - 42x + 49 =$$

RHS

$$\text{LHS} = \text{RHS}$$

$$\text{ii) LHS} = (9p - 5q)^2 + 180pq = 81p^2 - 90pq + 25q^2 + 180pq$$

$$= 81p^2 + 90pq + 25q^2$$

$$\text{RHS} = (9p + 5q)^2$$

$$= 81p^2 + 90pq + 25q^2$$

$$\text{LHS} = \text{RHS}$$

$$\text{(iii) LHS} = \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn$$

$$= \frac{16}{9}m^2 + \frac{9}{16}n^2 - 2mn + 2mn$$

$$= \frac{16}{9}m^2 + \frac{9}{16}n^2$$

$$= \text{RHS}$$

$$\text{LHS} = \text{RHS}$$

$$\text{iv) LHS} = (4pq + 3q)^2 - (4pq - 3q)^2$$

$$= 16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24pq^2 - 9q^2$$

$$= 48pq^2$$

$$= \text{RHS}$$

$$\text{LHS} = \text{RHS}$$

$$\text{v) LHS} = (a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a)$$

$$= a^2 - b^2 + b^2 - c^2 + c^2 - a^2$$

$$= 0$$

$$= \text{RHS}$$

14. Using identities, evaluate.

$$\text{(i) } 71^2$$

$$\text{(ii) } 99^2$$

$$\text{(iii) } 102^2$$

(iv) 998^2

(v) 5.2^2

(vi) 297×303

(vii) 78×82

(viii) 8.9^2

(ix) 10.5×9.5

Solution:

i) 71^2

$$= (70+1)^2$$

$$= 70^2 + 140 + 1^2$$

$$= 4900 + 140 + 1 =$$

$$5041$$

ii) 99^2

$$= (100 - 1)^2$$

$$= 100^2 - 200 + 1^2$$

$$= 10000 - 200 + 1 =$$

$$9801$$

iii) 102^2

$$= (100 + 2)^2$$

$$= 100^2 + 400 + 2^2 = 10000$$

$$+ 400 + 4 = 10404$$

iv) 998^2

$$= (1000 - 2)^2$$

$$= 1000^2 - 4000 + 2^2$$

$$= 1000000 - 4000 + 4 =$$

$$996004$$

v) 5.2^2

$$= (5 + 0.2)^2$$

$$= 5^2 + 2 + 0.2^2$$

$$= 25 + 2 + 0.04 = 27.04$$

$$\text{vi) } 297 \times 303$$

$$= (300 - 3)(300 + 3)$$

$$= 300^2 - 3^2$$

$$= 90000 - 9$$

$$= 89991 \text{ vii)}$$

$$78 \times 82$$

$$= (80 - 2)(80 + 2)$$

$$= 80^2 - 2^2$$

$$= 6400 - 4 =$$

$$6396$$

$$\text{viii) } 8.9^2$$

$$= (9 - 0.1)^2$$

$$= 9^2 - 1.8 + 0.1^2$$

$$= 81 - 1.8 + 0.01$$

$$= 79.21 \text{ ix) } 10.5$$

$$\times 9.5$$

$$= (10 + 0.5)(10 - 0.5)$$

$$= 10^2 - 0.5^2$$

$$= 100 - 0.25$$

$$= 99.75$$

15. Using $a^2 - b^2 = (a + b)(a - b)$, find

$$\text{(i) } 51^2 - 49^2$$

$$\text{(ii) } (1.02)^2 - (0.98)^2$$

$$\text{(iii) } 153^2 - 147^2$$

$$\text{(iv) } 12.1^2 - 7.9^2$$

Solution:

$$\text{i) } 51^2 - 49^2$$

$$= (51 + 49)(51 - 49) = 100 \times 2 = 200 \text{ ii)}$$

$$(1.02)^2 - (0.98)^2$$

$$= (1.02 + 0.98)(1.02 - 0.98) = 2 \times 0.04 = 0.08 \text{ iii)}$$

$$153^2 - 147^2$$

$$= (153 + 147)(153 - 147) = 300 \times 6 = 1800 \text{ iv)}$$

$$12.1^2 - 7.9^2$$

$$= (12.1 + 7.9)(12.1 - 7.9) = 20 \times 4.2 = 84$$