

CHAPTER 7

FACTORISATION

1. Find the common factors of the given terms.

(i) $12x, 36$

(ii) $2y, 22xy$

(iii) $14pq, 28p^2q^2$

(iv) $2x, 3x^2, 4$

(v) $6abc, 24ab^2, 12a^2b$

(vi) $16x^3, -4x^2, 32x$

(vii) $10pq, 20qr, 30rp$ (viii) $3x^2y^3, 10x^3y^2, 6x^2y^2z$ Solution:

(i) Factors of $12x$ and 36

$$12x = 2 \times 2 \times 3 \times x$$

$$36 = 2 \times 2 \times 3 \times 3$$

Common factors of $12x$ and 36 are $2, 2, 3$ and

$$, 2 \times 2 \times 3 = 12$$

(ii) Factors of $2y$ and $22xy$

$$2y = 2 \times y$$

$$22xy = 2 \times 11 \times x \times y$$

Common factors of $2y$ and $22xy$ are $2, y$ and

$$, 2 \times y = 2y$$

(iii) Factors of $14pq$ and $28p^2q^2$

$$14pq = 2 \times 7 \times p \times q$$

$$28p^2q^2 = 2 \times 2 \times 7 \times p \times p \times q \times q$$

Common factors of $14pq$ and $28p^2q^2$ are $2, 7, p, q$ and,

$$2 \times 7 \times p \times q = 14pq$$

(iv) Factors of $2x$, $3x^2$ and 4

$$2x = 2 \times x$$

$$3x^2 = 3 \times x \times x$$

$$4 = 2 \times 2$$

Common factors of $2x$, $3x^2$ and 4 is 1 .

(v) Factors of $6abc$, $24ab^2$ and $12a^2b$

$$6abc = 2 \times 3 \times a \times b \times c$$

$$24ab^2 = 2 \times 2 \times 2 \times 3 \times a \times b \times b$$

$$12a^2b = 2 \times 2 \times 3 \times a \times a \times b$$

Common factors of $6abc$, $24ab^2$ and $12a^2b$ are $2, 3, a, b$ and,

$$2 \times 3 \times a \times b = 6ab$$

(vi) Factors of $16x^3$, $-4x^2$ and $32x$

$$16x^3 = 2 \times 2 \times 2 \times 2 \times x \times x \times x$$

$$-4x^2 = -1 \times 2 \times 2 \times x \times x$$

$$32x = 2 \times 2 \times 2 \times 2 \times 2 \times x$$

Common factors of $16x^3$, $-4x^2$ and $32x$ are $2, 2, x$ and,

$$2 \times 2 \times x = 4x$$

(vii) Factors of $10pq$, $20qr$ and $30rp$

$$10pq = 2 \times 5 \times p \times q$$

$$20qr = 2 \times 2 \times 5 \times q \times r$$

$$30rp = 2 \times 3 \times 5 \times r \times p$$

Common factors of $10pq$, $20qr$ and $30rp$ are $2, 5$ and,

$$2 \times 5 = 10$$

(viii) Factors of $3x^2y^3$, $10x^3y^2$ and $6x^2y^2z$

$$3x^2y^3 = 3 \times x \times x \times y \times y \times y$$

$$10x^3y^2 = 2 \times 5 \times x \times x \times x \times y \times y$$

$$6x^2y^2z = 3 \times 2 \times x \times x \times y \times y \times z$$

Common factors of $3x^2y^3$, $10x^3y^2$ and $6x^2y^2z$ are x^2 , y^2 and,

$$x^2 \times y^2 = x^2y^2$$

2. Factorise the following expressions.

(i) $7x - 42$

(ii) $6p - 12q$

(iii) $7a^2 + 14a$

(iv) $-16z + 20z^3$

(v) $20l^2m + 30alm$

(vi) $5x^2y - 15xy^2$

(vii) $10a^2 - 15b^2 + 20c^2$

(viii) $-4a^2 + 4ab - 4ca$

(ix) $x^2yz + xy^2z + xyz^2$

(x) $ax^2y + bxy^2 + cxyz$

Solution:

(i) $7x = 7 \times x$

$42 = 2 \times 3 \times 7$

The common factor is 7

$\therefore 7x - 42 = (7 \times x) - (2 \times 3 \times 7) = 7(x - 6)$

(ii) $6p = 2 \times 3 \times p$

$12q = 2 \times 2 \times 3 \times q$

The common factors are 2 and 3

$\therefore 6p - 12q = (2 \times 3 \times p) - (2 \times 2 \times 3 \times q)$

$= 2 \times 3 [p - (2 \times q)]$

$= 6(p - 2q)$

(iii) $7a^2 = 7 \times a \times a$

$14a = 2 \times 7 \times a$

The common factors are 7 and a

$\therefore 7a^2 + 14a = (7 \times a \times a) + (2 \times 7 \times a)$

$= 7 \times a [a + 2] = 7a(a + 2)$

(iv) $16z = 2 \times 2 \times 2 \times 2 \times z$

$20z^3 = 2 \times 2 \times 5 \times z \times z \times z$

The common factors are 2, 2, and z.

$\therefore -16z + 20z^3 = -(2 \times 2 \times 2 \times 2 \times z) + (2 \times 2 \times 5 \times z \times z \times z)$

$= (2 \times 2 \times z) [-(2 \times 2) + (5 \times z \times z)]$

$= 4z(-4 + 5z^2)$

$$(v) \ 20l^2m = 2 \times 2 \times 5 \times l \times l \times m$$

$$30alm = 2 \times 3 \times 5 \times a \times l \times m$$

The common factors are 2, 5, l and m .

$$\begin{aligned} \therefore 20l^2m + 30alm &= (2 \times 2 \times 5 \times l \times l \times m) + (2 \times 3 \times 5 \times a \times l \times m) \\ &= (2 \times 5 \times l \times m) [(2 \times l) + (3 \times a)] \\ &= 10lm (2l + 3a) \end{aligned}$$

$$(vi) \ 5x^2y = 5 \times x \times x \times y$$

$$15xy^2 = 3 \times 5 \times x \times y \times y$$

The common factors are 5, x , and y .

$$\begin{aligned} \therefore 5x^2y - 15xy^2 &= (5 \times x \times x \times y) - (3 \times 5 \times x \times y \times y) \\ &= 5 \times x \times y [x - (3 \times y)] \\ &= 5xy (x - 3y) \end{aligned}$$

$$(vii) \ 10a^2 - 15b^2 + 20c^2$$

$$10a^2 = 2 \times 5 \times a \times a$$

$$-15b^2 = -1 \times 3 \times 5 \times b \times b$$

$$20c^2 = 2 \times 2 \times 5 \times c \times c$$

Common factor of $10a^2$, $15b^2$ and $20c^2$ is 5

$$10a^2 - 15b^2 + 20c^2 = 5(2a^2 - 3b^2 + 4c^2)$$

$$(viii) \ -4a^2 + 4ab - 4ca$$

$$-4a^2 = -1 \times 2 \times 2 \times a \times a$$

$$4ab = 2 \times 2 \times a \times b$$

$$-4ca = -1 \times 2 \times 2 \times c \times a$$

Common factor of $-4a^2$, $4ab$, $-4ca$ are 2, 2, a i.e. $4a$

So,

$$-4a^2 + 4ab - 4ca = 4a(-a + b - c) \quad (ix) \ x^2yz + xy^2z + xyz^2 = x \times x \times y \times z + x \times y \times y \times z + x \times y \times z \times z = x \times y \times z \times (x + y + z)$$

Common factor of x^2yz , xy^2z and xyz^2 are x , y , z i.e. xyz

$$\text{Now, } x^2yz + xy^2z + xyz^2 = xyz(x + y + z)$$

$$(x) \ ax^2y + bxy^2 + cxyz = ax^2y +$$

$$a \times x \times x \times y \quad bxy^2 = b \times x \times y \times y \quad cxyz =$$

$$c \times x \times y \times z$$

Common factors of a^2y , bxy^2 and $cxyz$ are xy

$$\text{Now, } ax^2y + bxy^2 + cxyz = xy(ax + by + cz)$$

3. Factorise.

(i) $x^2 + xy + 8x + 8y$

(ii) $15xy - 6x + 5y - 2$

(iii) $ax + bx - ay - by$

(iv) $15pq + 15 + 9q + 25p$

(v) $z - 7 + 7xy - xyz$ Solution:

$$\begin{aligned} \text{(i)} \quad x^2 + xy + 8x + 8y &= x \times x + x \times y + 8 \times x + 8 \times y \\ &= x(x + y) + 8(x + y) \\ &= (x + y)(x + 8) \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 15xy - 6x + 5y - 2 &= 3 \times 5 \times x \times y - 3 \times 2 \times x + 5xy - 2 \\ &= 3x(5y - 2) + 1(5y - 2) \\ &= (5y - 2)(3x + 1) \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad ax + bx - ay - by &= a \times x + b \times x - a \times y - b \times y \\ &= x(a + b) - y(a + b) \\ &= (a + b)(x - y) \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad 15pq + 15 + 9q + 25p &= 15pq + 9q + 25p + 15 \\ &= 3 \times 5 \times p \times q + 3 \times 3 \times q + 5 \times 5 \times p + 3 \times 5 \\ &= 3q(5p + 3) + 5(5p + 3) \\ &= (5p + 3)(3q + 5) \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad z - 7 + 7xy - xyz &= z - x \times y \times z - 7 + 7 \times x \times y \\ &= z(1 - xy) - 7(1 - xy) \\ &= (1 - xy)(z - 7) \end{aligned}$$

4. Factorise the following expressions.

(i) $a^2 + 8a + 16$

(ii) $p^2 - 10p + 25$

(iii) $25m^2+30m+9$

(iv) $49y^2+84yz+36z^2$

(v) $4x^2-8x+4$

(vi) $121b^2-88bc+16c^2$

(vii) $(l+m)^2-4lm$ (Hint: Expand $(l+m)^2$ first)

(viii) $a^4+2a^2b^2+b^4$

Solution:

(i) $a^2+8a+16$

$$= a^2+2 \times 4 \times a+4^2$$

$$= (a+4)^2$$

Using the identity $(x+y)^2 = x^2+2xy+y^2$

(ii) $p^2-10p+25$

$$= p^2-2 \times 5 \times p+5^2$$

$$= (p-5)^2$$

Using the identity $(x-y)^2 = x^2-2xy+y^2$

(iii) $25m^2+30m+9$

$$= (5m)^2+2 \times 5m \times 3+3^2$$

$$= (5m+3)^2$$

Using the identity $(x+y)^2 = x^2+2xy+y^2$

(iv) $49y^2+84yz+36z^2$

$$= (7y)^2+2 \times 7y \times 6z+(6z)^2$$

$$= (7y+6z)^2$$

Using the identity $(x+y)^2 = x^2+2xy+y^2$

(v) $4x^2-8x+4$

$$= (2x)^2-2 \times 4x+2^2$$

$$= (2x-2)^2$$

Using the identity $(x-y)^2 = x^2 - 2xy + y^2$

(vi) $121b^2 - 88bc + 16c^2$

$$= (11b)^2 - 2 \times 11b \times 4c + (4c)^2$$

$$= (11b - 4c)^2$$

Using the identity $(x-y)^2 = x^2 - 2xy + y^2$

(vii) $(l+m)^2 - 4lm$ (Hint: Expand $(l+m)^2$ first)

Expand $(l+m)^2$ using the identity $(x+y)^2 = x^2 + 2xy + y^2$

$$(l+m)^2 - 4lm = l^2 + m^2 + 2lm - 4lm$$

$$= l^2 + m^2 - 2lm$$

$$= (l-m)^2$$

Using the identity $(x-y)^2 = x^2 - 2xy + y^2$

(viii) $a^4 + 2a^2b^2 + b^4$

$$= (a^2)^2 + 2 \times a^2 \times b^2 + (b^2)^2$$

$$= (a^2 + b^2)^2$$

Using the identity $(x+y)^2 = x^2 + 2xy + y^2$

5. Factorise.

(i) $4p^2 - 9q^2$

(ii) $63a^2 - 112b^2$

(iii) $49x^2 - 36$

(iv) $16x^5 - 144x^3$ differ

(v) $(l+m)^2 - (l-m)^2$

(vi) $9x^2y^2 - 16$

(vii) $(x^2 - 2xy + y^2) - z^2$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

Solution:

(i) $4p^2 - 9q^2$

$$= (2p)^2 - (3q)^2$$

$$= (2p-3q)(2p+3q)$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$

$$(ii) \quad 63a^2 - 112b^2$$

$$= 7(9a^2 - 16b^2)$$

$$= 7((3a)^2 - (4b)^2)$$

$$= 7(3a+4b)(3a-4b)$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$

$$(iii) \quad 49x^2 - 36$$

$$= (7x)^2 - 6^2$$

$$= (7x+6)(7x-6)$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$

$$(iv) \quad 16x^5 - 144x^3$$

$$= 16x^3(x^2 - 9)$$

$$= 16x^3(x^2 - 9)$$

$$= 16x^3(x-3)(x+3)$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$

$$(v) \quad (l+m)^2 - (l-m)^2$$

$$= \{(l+m) - (l-m)\} \{(l+m) + (l-m)\}$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$

$$= (l+m-l+m)(l+m+l-m)$$

$$= (2m)(2l)$$

$$= 4ml$$

$$(vi) \quad 9x^2y^2 - 16$$

$$= (3xy)^2 - 4^2$$

$$= (3xy-4)(3xy+4)$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$

$$(vii) (x^2 - 2xy + y^2) - z^2$$

$$= (x-y)^2 - z^2$$

Using the identity $(x-y)^2 = x^2 - 2xy + y^2$

$$= \{(x-y) - z\} \{(x-y) + z\}$$

$$= (x-y-z)(x-y+z)$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$

$$(viii) 25a^2 - 4b^2 + 28bc - 49c^2$$

$$= 25a^2 - (4b^2 - 28bc + 49c^2)$$

$$= (5a)^2 - \{(2b)^2 - 2(2b)(7c) + (7c)^2\}$$

$$= (5a)^2 - (2b - 7c)^2$$

Using the identity $x^2 - y^2 = (x+y)(x-y)$, we have

$$= (5a + 2b - 7c)(5a - 2b + 7c)$$

6. Factorise the expressions.

$$(i) \quad ax^2 + bx$$

$$(ii) \quad 7p^2 + 21q^2$$

$$(iii) \quad 2x^3 + 2xy^2 + 2xz^2$$

$$(iv) \quad am^2 + bm^2 + bn^2 + an^2$$

$$(v) \quad (lm + l) + m + 1$$

$$(vi) \quad y(y+z) + 9(y+z)$$

$$(vii) \quad 5y^2 - 20y - 8z + 2yz$$

$$(viii) \quad 10ab + 4a + 5b + 2$$

$$(ix) \quad 6xy - 4y + 6 - 9x \text{ Solution:}$$

$$(i) \quad ax^2 + bx = x(ax + b)$$

$$(ii) \quad 7p^2 + 21q^2 = 7(p^2 + 3q^2)$$

$$(iii) \quad 2x^3 + 2xy^2 + 2xz^2 = 2x(x^2 + y^2 + z^2)$$

$$(iv) \quad am^2+bm^2+bn^2+an^2 = m^2(a+b)+n^2(a+b) = (a+b)(m^2+n^2)$$

$$(v) \quad (lm+l)+m+1 = lm+m+l+1 = m(l+1)+(l+1) = (m+1)(l+1)$$

$$(vi) \quad y(y+z)+9(y+z) = (y+9)(y+z)$$

$$(vii) \quad 5y^2-20y-8z+2yz = 5y(y-4)+2z(y-4) = (y-4)(5y+2z)$$

$$(viii) \quad 10ab+4a+5b+2 = 5b(2a+1)+2(2a+1) = (2a+1)(5b+2) \quad (ix) \quad 6xy-4y+6-9x = 6xy-9x-4y+6 = 3x(2y-3)-2(2y-3) = (2y-3)(3x-2)$$

7. Factorise.

$$(i) \quad a^4-b^4$$

$$(ii) \quad p^4-81$$

$$(iii) \quad x^4-(y+z)^4$$

$$(iv) \quad x^4-(x-z)^4 \quad (v) \quad a^4-2a^2b^2+b^4$$

Solution:

$$(i) \quad a^4-b^4$$

$$= (a^2)^2-(b^2)^2$$

$$= (a^2-b^2)(a^2+b^2)$$

$$= (a-b)(a+b)(a^2+b^2)$$

$$(ii) \quad p^4-81$$

$$= (p^2)^2-(9)^2$$

$$= (p^2-9)(p^2+9)$$

$$= (p^2-3^2)(p^2+9)$$

$$= (p-3)(p+3)(p^2+9)$$

$$(iii) \quad x^4-(y+z)^4 = (x^2)^2-[(y+z)^2]^2$$

$$= \{x^2-(y+z)^2\} \{x^2+(y+z)^2\}$$

$$= \{(x-(y+z))(x+(y+z))\} \{x^2+(y+z)^2\}$$

$$= (x-y-z)(x+y+z) \{x^2+(y+z)^2\}$$

$$(iv) \quad x^4-(x-z)^4 = (x^2)^2-\{(x-z)^2\}^2$$

$$= \{x^2-(x-z)^2\} \{x^2+(x-z)^2\}$$

$$= \{x-(x-z)\} \{x+(x-z)\} \{x^2+(x-z)^2\}$$

$$= z(2x-z)(x^2+x^2-2xz+z^2)$$

$$= z(2x-z)(2x^2-2xz+z^2)$$

$$(v) \quad a^4-2a^2b^2+b^4 = (a^2)^2-2a^2b^2+(b^2)^2$$

$$= (a^2-b^2)^2$$

$$= ((a-b)(a+b))^2$$

$$= (a-b)^2 (a+b)^2$$

8. Factorise the following expressions.

(i) p^2+6p+8

(ii) $q^2-10q+21$

(iii) $p^2+6p-16$ Solution:

(i) p^2+6p+8

We observed that $8 = 4 \times 2$ and $4+2 = 6$

p^2+6p+8 can be written as $p^2+2p+4p+8$

Taking Common terms, we get p^2+6p+8

$$= p^2+2p+4p+8 = p(p+2)+4(p+2) \text{ Again,}$$

$p+2$ is common in both the terms.

$$= (p+2)(p+4)$$

This implies that $p^2+6p+8 = (p+2)(p+4)$

(ii) $q^2-10q+21$

We observed that $21 = -7 \times -3$ and $-7+(-3) = -10$

$$q^2-10q+21 = q^2-3q-7q+21 = q(q-3)-7(q-3)$$

$$= (q-7)(q-3)$$

This implies that $q^2-10q+21 = (q-7)(q-3)$

(iii) $p^2+6p-16$

We observed that $-16 = -2 \times 8$ and $8 + (-2) = 6$ $p^2 + 6p - 16$

$$= p^2 - 2p + 8p - 16$$

$$= p(p-2) + 8(p-2)$$

$$= (p+8)(p-2)$$

$$\text{So, } p^2 + 6p - 16 = (p+8)(p-2)$$

9. Carry out the following divisions.

(i) $28x^4 \div 56x$

(ii) $-36y^3 \div 9y^2$

(iii) $66pq^2r^3 \div 11qr^2$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$

(v) $12a^8b^8 \div (-6a^6b^4)$

Solution:

$$(i) 28x^4 = 2 \times 2 \times 7 \times x \times x \times x \times x$$

$$56x = 2 \times 2 \times 2 \times 7 \times x$$

$$28x^4 \div 56x = \frac{2 \times 2 \times 7 \times x \times x \times x \times x}{2 \times 2 \times 2 \times 7 \times x} = \frac{x^3}{2} = \frac{1}{2}x^3$$

$$(ii) -36y^3 \div 9y^2 = \frac{-2 \times 2 \times 3 \times 3 \times y \times y \times y}{3 \times 3 \times y \times y} = -4y$$

$$(iii) 66pq^2r^3 \div 11qr^2 = \frac{2 \times 3 \times 11 \times p \times q \times q \times r \times r \times r}{11 \times q \times r \times r} = 6pqr$$

$$(iv) 34x^3y^3z^3 \div 51xy^2z^3 = \frac{2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times z}{3 \times 17 \times x \times y \times y \times z \times z \times z} = \frac{2}{3}x^2y$$

$$(v) 12a^8b^8 \div (-6a^6b^4) = \frac{2 \times 2 \times 3 \times a^8 \times b^8}{-2 \times 3 \times a^6 \times b^4} = -2a^2b^4$$

10. Divide the given polynomial by the given monomial.

(i) $(5x^2 - 6x) \div 3x$

(ii) $(3y^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$

(iv) $(x^3 + 2x^2 + 3x) \div 2x$

(v) $(p^3q^6 - p^6q^3) \div p^3q^3$

Solution:

$$(i) 5x^2 - 6x = x(5x - 6)$$

$$(5x^2 - 6x) \div 3x = \frac{x(5x - 6)}{3x} = \frac{1}{3}(5x - 6)$$

$$(ii) 3y^8 - 4y^6 + 5y^4 = y^4(3y^4 - 4y^2 + 5)$$

$$(3y^8 - 4y^6 + 5y^4) \div y^4 = \frac{y^4(3y^4 - 4y^2 + 5)}{y^4} = 3y^4 - 4y^2 + 5$$

$$(iii) 8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) = 8x^2y^2z^2(x + y + z)$$

$$8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2 = \frac{8x^2y^2z^2(x + y + z)}{4x^2y^2z^2} = 2(x + y + z)$$

$$(iv) x^3 + 2x^2 + 3x = x(x^2 + 2x + 3)$$

$$(x^3 + 2x^2 + 3x) \div 2x = \frac{x(x^2 + 2x + 3)}{2x} = \frac{1}{2}(x^2 + 2x + 3)$$

$$(v) p^3q^6 - p^6q^3 = p^3q^3(q^3 - p^3)$$

$$(p^3q^6 - p^6q^3) \div p^3q^3 = \frac{p^3q^3(q^3 - p^3)}{p^3q^3} = q^3 - p^3$$

11. Work out the following divisions.

(i) $(10x - 25) \div 5$

(ii) $(10x - 25) \div (2x - 5)$

(iii) $10y(6y + 21) \div 5(2y + 7)$

(iv) $9x^2y^2(3z - 24) \div 27xy(z - 8)$

$$(v) 96abc(3a-12)(5b-30) \div 144(a-4)(b-6)$$

Solution:

$$(i) (10x-25) \div 5 = 5(2x-5)/5 = 2x-5$$

$$(ii) (10x-25) \div (2x-5) = 5(2x-5)/(2x-5) = 5$$

$$(iii) 10y(6y+21) \div 5(2y+7) = 10y \times 3(2y+7)/5(2y+7) = 6y$$

$$(iv) 9x^2y^2(3z-24) \div 27xy(z-8) = 9x^2y^2 \times 3(z-8)/27xy(z-8) = xy$$

$$(v) \underline{96abc(3a-12)(5b-30)} \div 144(a-4)(b-6) = \frac{96abc \times 3(a-4) \times 5(b-6)}{144(a-4)(b-6)} = 10abc$$

12. Divide as directed.

$$(i) 5(2x+1)(3x+5) \div (2x+1)$$

$$(ii) 26xy(x+5)(y-4) \div 13x(y-4)$$

$$(iii) 52pqr(p+q)(q+r)(r+p) \div 104pq(q+r)(r+p)$$

$$(iv) 20(y+4)(y^2+5y+3) \div 5(y+4)$$

$$(v) x(x+1)(x+2)(x+3) \div x(x+1) \text{ Solution:}$$

$$(i) 5(2x+1)(3x+5) \div (2x+1) = \frac{5(2x+1)(3x+5)}{(2x+1)} \\ = 5(3x+5)$$

$$(ii) 26xy(x+5)(y-4) \div 13x(y-4) = \frac{2 \times 13 \times xy(x+5)(y-4)}{13x(y-4)} \\ = 2y(x+5)$$

$$(iii) 52pqr(p+q)(q+r)(r+p) \div 104pq(q+r)(r+p) \\ = \frac{2 \times 2 \times 13 \times p \times q \times r \times (p+q) \times (q+r) \times (r+p)}{2 \times 2 \times 2 \times 13 \times p \times q \times (q+r) \times (r+p)} \\ = \frac{1}{2}r(p+q)$$

$$(iv) 20(y+4)(y^2+5y+3) \div 5(y+4) = \frac{2 \times 2 \times 5 \times (y+4)(y^2+5y+3)}{5(y+4)} \\ = 4(y^2+5y+3)$$

$$(v) x(x+1)(x+2)(x+3) \div x(x+1) = \frac{x(x+1)(x+2)(x+3)}{x(x+1)} \\ = (x+2)(x+3)$$

13. Factorise the expressions and divide them as directed.

(i) $(y^2+7y+10) \div (y+5)$

(ii) $(m^2-14m-32) \div (m+2)$

(iii) $(5p^2-25p+20) \div (p-1)$

(iv) $4yz(z^2+6z-16) \div 2y(z+8)$

(v) $5pq(p^2-q^2) \div 2p(p+q)$

(vi) $12xy(9x^2-16y^2) \div 4xy(3x+4y)$

(vii) $39y^3(50y^2-98) \div 26y^2(5y+7)$

Solution:

(i) $(y^2+7y+10) \div (y+5)$

First, solve the equation $(y^2+7y+10)$

$$(y^2+7y+10) = y^2+2y+5y+10 = y(y+2)+5(y+2) = (y+2)(y+5)$$

$$\text{Now, } (y^2+7y+10) \div (y+5) = (y+2)(y+5)/(y+5) = y+2$$

(ii) $(m^2-14m-32) \div (m+2)$

Solve for $m^2-14m-32$, we have $m^2-14m-32 = m^2+2m-16m-32 =$

$$m(m+2)-16(m+2) = (m-16)(m+2)$$

$$\text{Now, } (m^2-14m-32) \div (m+2) = (m-16)(m+2)/(m+2) = m-16$$

(iii) $(5p^2-25p+20) \div (p-1)$

Step 1: Take 5 common from the equation, $5p^2-25p+20$, we get

$$5p^2-25p+20 = 5(p^2-5p+4) \text{ Step 2:}$$

Factorise p^2-5p+4 $p^2-5p+4 = p^2-p-$

$$4p+4 = (p-1)(p-4)$$

Step 3: Solve original equation

$$(5p^2-25p+20) \div (p-1) = 5(p-1)(p-4)/(p-1) = 5(p-4)$$

(iv) $4yz(z^2 + 6z-16) \div 2y(z+8)$ Factorising $z^2+6z-16$, $z^2+6z-16 = z^2-2z+8z-16 = (z-2)(z+8)$

$$\text{Now, } 4yz(z^2+6z-16) \div 2y(z+8) = 4yz(z-2)(z+8)/2y(z+8) = 2z(z-2)$$

(v) $5pq(p^2-q^2) \div 2p(p+q)$ p^2-q^2 can be written as $(p-q)(p+q)$ using the identity.

$$5pq(p^2-q^2) \div 2p(p+q) = 5pq(p-q)(p+q)/2p(p+q) = 5q(p-q)/2$$

(vi) $12xy(9x^2-16y^2) \div 4xy(3x+4y)$

Factorising $9x^2-16y^2$, we have

$$9x^2-16y^2 = (3x)^2-(4y)^2 = (3x+4y)(3x-4y) \text{ using the identity } p^2-q^2 = (p-q)(p+q)$$

$$\text{Now, } 12xy(9x^2-16y^2) \div 4xy(3x+4y) = 12xy(3x+4y)(3x-4y) / 4xy(3x+4y) = 3(3x-4y)$$

(vii) $39y^3(50y^2-98) \div 26y^2(5y+7)$ st solve for $50y^2-98$, we have

$$50y^2-98 = 2(25y^2-49) = 2((5y)^2-7^2) = 2(5y-7)(5y+7)$$

$$\text{Now, } 39y^3(50y^2-98) \div 26y^2(5y+7) =$$

$$\frac{3 \times 13 \times y^3 \times 2(5y-7)(5y+7)}{2 \times 13 \times y^2(5y+7)} = 3y(5y-7)$$